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AN INVESTIGATION OF THE  
 $p^{31}(d,p)p^{32}$  REACTION

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DANIEL PIRAINO

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AN INVESTIGATION OF THE  $P^{31}(d,p)P^{32}$  REACTION

by

Daniel Piraino

B. S., University of Idaho  
(1951)

Submitted in Partial Fulfillment of the  
Requirements for the Degree of

Master of Science

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 1958





# AN INVESTIGATION OF THE $P^{31}(d,p)P^{32}$ REACTION

by

Daniel Piraino  
Lieutenant, U. S. Navy

Submitted to the Department of Physics on May 26, 1958  
in partial fulfillment of the requirements for the degree of

Master of Science

## ABSTRACT

The MIT-ONR electrostatic accelerator and broad-range spectrograph have been employed to investigate the  $P^{31}(d,p)P^{32}$  reaction. Thin targets of silver pyrophosphate evaporated onto Formvar films were bombarded with 6-Mev deuterons. The proton groups corresponding to excitation of the ground state of  $P^{32}$  and of the fifty-two excited levels were analyzed at laboratory angles of 30, 50, 70, and 90 degrees. The ground-state Q-value was found to be  $5.709 \pm 0.010$  Mev in good agreement with results by other experimenters. Several levels, which were obscured by contaminant groups in the previous work on this nucleus, were identified, as well as many levels not previously resolved.

Because of the relatively large amounts of sodium in the evaporated material, it was possible to confirm the ground-state Q-value of the  $Na^{23}(d,p)Na^{24}$  reaction. Proton groups corresponding to transitions to the ground state and nineteen excited levels in  $Na^{24}$  were found in this work.

Thesis Supervisor: W. W. Duechner

Title: Professor of Physics

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The author wishes to express his appreciation and thanks to the members of the High Voltage Laboratory for their assistance and cooperation, without which this work could not have been accomplished.

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Finally, I wish to thank Mrs. Mary E. White for the excellent preparation of the manuscript.

MEMORANDUM

The author wishes to express his appreciation and thanks to the members of the High Voltage Laboratory for their assistance and cooperation, without which this work could not have been accomplished.

I am particularly indebted to Dr. H. A. Fiske of the University of Chicago for his guidance and instruction during the early part of the experimental work. Professor W. A. Bushong was a most helpful and patient advisor, as were Professors H. A. Fiske, H. A. Fiske, and Dr. A. J. Smith of the University of Chicago.

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## I. Introduction

The MIT-ONR electrostatic accelerator used in conjunction with the broad-range spectrograph is particularly well suited to the determination of nuclear energy levels. In this work, the equipment is used to determine the energy levels of  $P^{32}$  by analysis of the proton groups from the  $P^{31}(d,p)P^{32}$  reaction.

According to the shell model<sup>1</sup>, the nucleus  $P^{32}$  has a single proton hole in the  $s_{1/2}$  shell and a single neutron in the  $d_{3/2}$  shell. It is therefore amenable to treatment by the method of Goldstein and Talmi.<sup>2</sup> These authors have developed a set of equations for determining the binding energies of nuclei such as  $P^{32}$  in terms of various nuclear parameters, including the nuclear spins of the ground state and of the excited levels. To provide a check for these calculations, it is desirable to have information on the energy levels and spins of the levels in the nucleus. The energy levels are determined by methods described in this work. A limit can be set on the spins of the levels by employing the stripping theory of Butler.<sup>3</sup> Butler derives an expression for the cross section of the  $(d,p)$  reaction as a function of the reaction angle, with  $\ell_n$ , the momentum of the captured neutron, as a parameter. By measuring the total number of particles at various angles in a proton group corresponding to a particular nuclear state, the experimental variation of reaction cross section can be determined. This is then compared with the theoretical curve from the Butler theory to determine the value of  $\ell_n$  which gives the best fit to the data. In this way, limits can be set on the spin of the level in question in accordance with the relation<sup>4</sup>

## I. Introduction

The first step in the analysis of the data is to determine the relationship between the energy levels and the wave function. The energy levels are determined by the wave function, and the wave function is determined by the energy levels. This is a self-consistent problem, and it is solved by the method of successive approximations. The first approximation is to assume that the energy levels are independent of the wave function. This is a reasonable assumption for the first approximation, but it is not true for the higher approximations. The second approximation is to assume that the energy levels are dependent on the wave function. This is a more realistic assumption, but it is still not true for the higher approximations. The third approximation is to assume that the energy levels are dependent on the wave function in a non-linear way. This is the most realistic assumption, but it is also the most difficult to solve. The method of successive approximations is used to solve this problem. The first approximation is to assume that the energy levels are independent of the wave function. This is a reasonable assumption for the first approximation, but it is not true for the higher approximations. The second approximation is to assume that the energy levels are dependent on the wave function. This is a more realistic assumption, but it is still not true for the higher approximations. The third approximation is to assume that the energy levels are dependent on the wave function in a non-linear way. This is the most realistic assumption, but it is also the most difficult to solve. The method of successive approximations is used to solve this problem.

$$\min \text{ of } \left| \pm J_A \pm l_n \pm \frac{1}{2} \right| \leq J_B \leq J_A + l_n + \frac{1}{2}$$

where

$J_A$  = spin of target nucleus

$J_B$  = spin of residual nucleus

$l_n$  = angular momentum of captured neutron

As the first step in obtaining the necessary data, this work was aimed at identifying and measuring the energy of the excited states of  $P^{32}$  from an analysis of the  $P^{31}(d,p)P^{32}$  reaction. An accurate Q-value for the ground-state transition is also obtained in this analysis.





## II. Apparatus

This investigation was carried out using the MIT-OMR electrostatic accelerator and the associated deflecting magnet, collimating slit system, target chamber, and broad-range spectrograph.<sup>5,6</sup>

### Analyzing Magnet

A cut-away drawing depicting the salient features of the accelerator is shown in Figure 1. The energy of the charged-particle beam and the spread in energy are defined by a collimating slit system and the deflecting magnet. The latter acts as a momentum filter. The momenta of particles which can pass through the magnet gap are determined by the setting of the magnetic field strength, which can be accurately measured as described below. The slit system controls the allowed energy spread in the incident beam, according to the relation<sup>7</sup>

$$R = \frac{a_0}{2\omega'} (M + 1)$$

where

R = energy resolution

$a_0$  = radius of curvature of rays

$\omega' = M\omega$

$\omega$  = width of entrance slit

M = magnification.

## II. Apparatus

This investigation was carried out using the RT-200 electron microscope and the associated deflection system, collector, and slit system, image camera, and photo-electronic system.

### Analyzing System

A cut-away diagram depicting the relative features of the apparatus is shown in Figure 1. The theory of the apparatus and its operation is defined by a collection of equations and the deflection system. The latter was as a secondary filter. The number of particles which can pass through the system was determined by the setting of the magnetic field strength which can be accurately measured as described below. The slit system controls the electron energy spread in the incident beam, according to the relation

$$E = \frac{h^2}{2me} (k + 1)$$

where

$E$  = energy resolution

$h$  = radius of curvature of rays

$m$  = mass

$k$  = width of electron slit

$e$  = electron charge

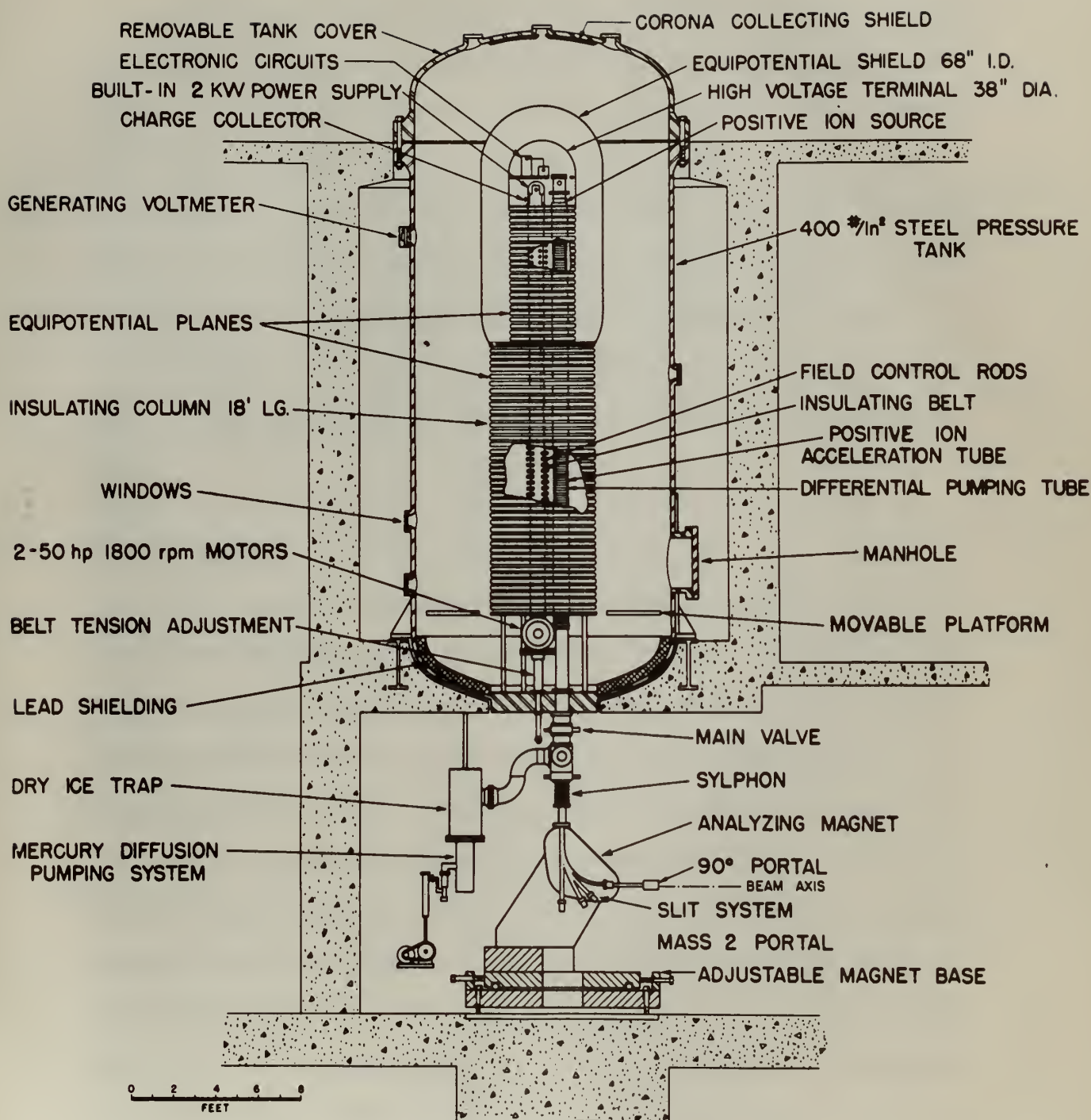


Figure 1







For the magnet in use at M.I.T.,  $M \approx 0.6$ ,  $a_0 \approx 60$  cm, so that a slit width of 1 mm gives an energy resolution of about 1:800. The spread in the input energy is further controlled by a corona current stabilizer that uses a portion of the beam current to control the current to the terminal. After passing through the deflector magnet, where its direction is rotated through 90 degrees, the beam is focused on a set of defining slits placed 185 cm from the exit face of the magnet. The beam current collected on these insulated slit jaws is used to provide the corona current stabilization mentioned above.

#### Target Chamber

The particle beam next enters the target chamber in which are placed the thin targets to be bombarded. It has been observed that, when targets are exposed to a particle beam, contaminants, particularly carbon, build up on the surface of the target illuminated by the beam. This build-up causes a broadening of the output particle peaks because of energy straggling and introduces an error into the energy determinations. To minimize this effect and also to preclude fracturing of the fragile targets, the target frames are mounted in a rotating mechanism. This consists of a small D. C. motor mounted on the lid of the target chamber. A shaft is mounted eccentrically on the rotor, and this in turn holds the target frame. A circular motion of the target in a plane perpendicular to the incident beam

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1900

The particle beam used within the target chamber is within one  
inches of the target to be irradiated. It has been observed that  
when targets are exposed to a particle beam, contaminants, particu-  
larly carbon, build up on the surface of the target illuminated by  
the beam. This build-up causes a broadening of the output particle  
beam because of energy straggling and introduces an error into the  
energy determinations. To minimize this effect and also to provide  
fracturing of the fragile targets, the target frames are secured in  
a rotating mechanism. This consists of a small 1/2 in. motor mounted  
on the rim of the target support. A shaft is secured eccentrically  
on the motor, and this in turn rotates the target frame. A nitrogen  
jet of the target is a fine counterstream to the incident beam

is thus produced. This causes the beam spot to illuminate the target over a ring-shaped area on the target of mean diameter about 1 cm.

### Broad-range Spectrograph

The particles emerging from the target chamber into the acceptance angle of the spectrograph (Figure 2) are deflected by the spectrograph's magnetic field with radii of curvature proportional to their momenta. Placed in the hyperbolic focal surface of the spectrograph are three 2 x 10-inch Eastman NTA 25-micron photographic plates that record the emergent particles. The plates are held rigidly in a specially constructed plateholder that constrains them to conform to the focal surface. Prior to loading in the spectrograph, the plates are indexed by shining light through a series of slits whose separations are very accurately known. These index marks serve as reference points for measuring distance along the plates when counting the particle tracks. The spectrograph can be rotated about a vertical axis through angles from zero to 130 degrees with respect to the direction of the incident beam. The angle of observation can be determined with an error of less than 10 minutes of arc. The solid angle of the spectrograph is independent of the angle of observation for the usual setting of the collimating slits. It is defined by an entrance aperture and by an 8 mm slit in front of the focal surface. The aperture angle is adjustable, but is normally used with a half angle of  $2\frac{1}{2}$  degrees. The solid angle on the





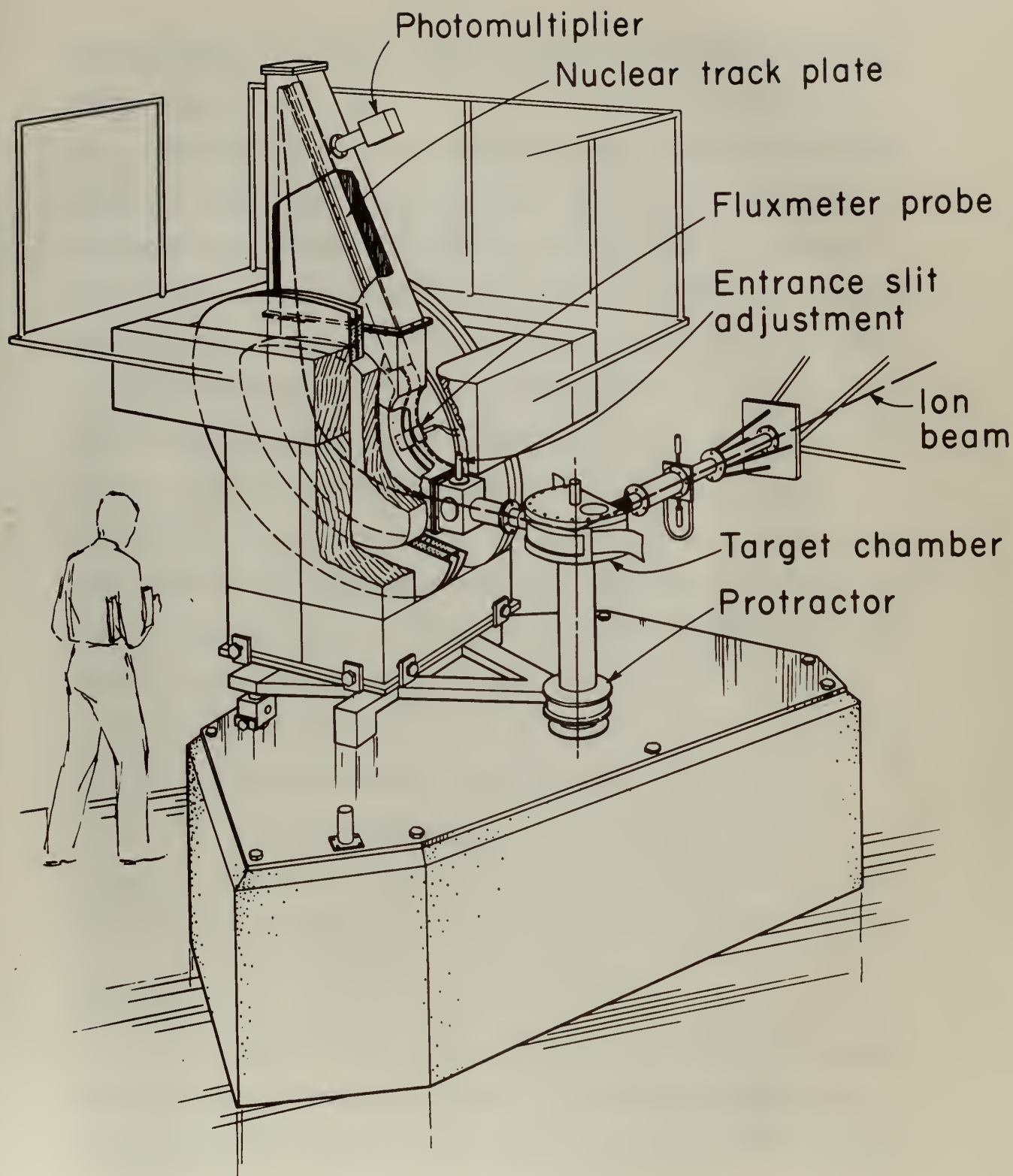


Figure 2



focal surface is about  $3.4 \times 10^{-4}$  steradians at a plate distance of 52 cm.

In order to have a correlation between the position of charged-particle groups along the photographic plate and their momentum, it is necessary to calibrate the spectrograph with a known standard. Alpha particles from the radioactive isotope  $\text{Po}^{210}$  (RaF) are used for this purpose. A silver wire is coated with a thin layer of polonium by dipping it into a solution of a polonium salt for about 10 minutes. The wire is then placed in the target chamber, and its position is fixed to correspond with the position of the beam spot from the accelerator. The alpha particles are allowed to enter the spectrograph where they are deflected onto the photographic plates. This is done for various values of the spectrograph magnetic field strength. The distances along the plate at which the one-third height of the alpha-particle groups is found are then plotted against their radii of curvature, as determined by the known value of their momenta ( $B\rho = 331.59$  kilogauss-centimeters) and the magnetic induction  $B$ . The validity of using the distance to the one-third height on the high-energy side of the peak is discussed in detail by Buechner et al.<sup>8</sup>

### Fluxmeters

The magnetic fields in the deflector and spectrograph magnets are measured and continuously monitored by fluxmeters employing a nuclear magnetic resonance technique. An aqueous solution of lithium



focal surface is kept 0.5 mm. separation at a plate distance of 52 cm.

In order to have a correlation between the position of charged particles across the photographic plate and their momentum, it is necessary to calibrate the spectrograph with a known standard. Alpha particles from the radioactive isotope  $^{210}\text{Po}$  are used for this purpose. A silver wire is coated with a thin layer of polonium by dipping it into a solution of a polonium salt for about 10 minutes. The wire is then placed in the spectrograph, and the position is fixed to correspond with the position of the beam spot from the accelerator. The alpha particles are allowed to enter the spectrograph where they are collected onto the photographic plates. This is done for various values of the spectrograph magnetic field strength. The distances where the plates are placed are one-third height of the alpha-particle proton is found and then plotted against their values of curvature, as determined by the known value of their momenta ( $p = 3.1 \times 10^{-21}$  g cm/sec) and the magnetic induction B. The validity of using the distance as the one-third height on the high-energy side of the peak is discussed in detail by Henschel et al.

Discussion

The magnetic fields in the detector and spectrograph magnets are measured and continuously monitored by fluxmeter readings. The present technique requires a constant calibration of the fluxmeters.



chloride contained in a small glass capsule is positioned between the pole faces of the magnet. The lithium nuclei will precess in the magnetic field with Larmor frequency

$$\nu = \frac{\mu_z H}{I h}$$

where

$\mu_z$  = the component of magnetic moment along H

H = magnetic field intensity

I = angular momentum of nucleus

h = Planck's constant.

A radio-frequency field of frequency F is applied to the capsule with its magnetic vector at right angles to H. At the resonance frequency  $F = \nu$ , changes occur in the orientation of the nuclear moments corresponding to transitions between the magnetic substates in accordance with the relation<sup>9</sup>

$$E = m_l h \nu$$

where

$m_l = I \cos \beta$  is the magnetic quantum number for the projection of I along H

E = kinetic energy of Larmor precession.

This absorption of energy by the lithium nuclei causes a relatively large depression of the amplitude of the radio-frequency

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relatively large amounts of the following

signal applied to the capsule. This signal is applied to the vertical deflection plates of a cathode-ray oscilloscope. The damping of the signal at resonance is observed as a depression of the sweep on the face of the oscilloscope, and the magnet current is adjusted to keep the "pip" centered. This procedure allows the field strength to be determined to within the accuracy of measurement of the radio-frequency. The latter can be measured with an accuracy of 1 part in 100,000. A secondary frequency standard, which is checked periodically with station WWV, is used to determine the frequency accurately.

#### Alterations

During the period covered by this investigation, several changes were made in the accelerator. These were aimed at increasing the maximum operating voltage and beam current attainable.

The thyrites, which had been used to distribute the voltage along the length of the column, were replaced with resistors. These thyrites served to maintain a uniform voltage gradient throughout the length of the accelerator column. However, the thyrites originally installed for this purpose had a nonlinear characteristic such that the voltage across them saturated at a predetermined value of current. Therefore, based on previous experience and also on the experiences reported by other Van de Graaff groups, it was decided to install a linear resistor manufactured by the S. S. White Dental Manufacturing Company. This change resulted in several problems.



signal needed to be known. This signal is applied to the  
first detection stage of a cathode-ray oscilloscope. The design  
of the signal at reception is shown as a derivation of the wave  
on the face of the oscilloscope, and the same circuit is applied  
to keep the "pin" constant. This procedure allows the time signals  
to be determined to within the accuracy of measurement of the radio-  
frequency. The latter can be measured with an accuracy of 1 part in  
100,000. A secondary frequency standard, which is constant within  
daily with accuracy, is used to determine the frequency standard.

#### Illustration

During the period covered by this investigation, several  
changes were made in the oscillator. These were aimed at increas-  
ing the number of cycles of voltage and hence output efficiency.  
The oscillator, which had been used to drive the relay  
along the length of the relay, was replaced with a relay. These  
cycles were used to maintain a constant voltage gradient throughout the  
length of the accelerator column. However, the cycles originally  
installed for this purpose had a constant electromotive force and  
the voltage across them was used as a reference value of current.  
Therefore, based on previous experience and also on the experience  
reported by other men in similar work, it was decided to install a  
linear resistor connected to the A.C. source which would  
compensate. This change resulted in several problems.

The first of these was the effect of ionization of the nitrogen gas used to insulate the generator. This ionization was caused by x-rays produced by electrons accelerated up the accelerator tube and stopped near the positive ion source at the top of the tube. This ionization caused a lowering of the potential of the intermediate shell, since the ion current from this shell to the walls of the tank was greater than that from the terminal to the shell because of the larger volume of gas involved. In attempting to bring the voltage back up, excessive charging currents were required on the accelerator belt, and this led to instability of the generator. The situation was remedied by construction of a 2-inch thick lead shield surrounding the ion source. This shield effectively attenuated the radiation produced by electron loading and thereby sufficiently reduced the ionization currents.

A second effect noted was the difficulty of stabilizing the generator voltage. The stabilizer presently in use controls the corona current to the intermediate shell which is connected to an equipotential plane two-thirds of the distance from the lower end of the generator. Present plans call for removal of this shell so that the stabilizer can control corona current to the terminal and thus have a stronger effect on the total generator voltage.

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### Target Preparation

The targets used in this work were prepared by evaporation of silver pyrophosphate onto thin Formvar films held on copper frames. The evaporation was carried out in a vacuum of about  $10^{-3}$  mm of mercury. The material to be evaporated was carefully weighed (100 milligrams was used) and placed in a tantalum boat, which was heated by the passage of an electric current. The silver pyrophosphate evaporated from the boat and subsequently condensed onto the Formvar films that were placed at distances of 2 to 6 inches from the boat. The targets were of the order of 5- to 10-kev thick for 6-Mev incident deuterons. This thickness was determined by use of an alpha-particle thickness gauge<sup>10</sup> which measures the thickness in terms of an equivalent thickness of air. This figure can then be used to determine the thickness in terms of energy loss of incident particles.

Large Deposits

The large coal in this area was removed by excavation of  
river deposits with heavy iron ore on the river.

The excavation was done out in a series of about 10-15 ft.

depth. The material in the excavation was mostly washed (100

percent) and used in a concrete base, which was covered

by the passage of an electric current. The river deposits

covered from the coal and subsequently contained with the river

river from the point to the river of 1 to 2 miles from the river.

The large size of the river of 1 to 10-15 miles for 5-10

inches diameter. The deposits were deposited by way of an

electricity in the river. The deposits were deposited in

series of an electric current of 10-15 miles from the river

and in between the deposits in series of every foot of deposit

partially.

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### III. Experimental Procedure

#### The Q-Equation

The type of reaction studied in this work is illustrated in Figure 3 (laboratory system).

The laws of conservation of energy and momentum require

$$E_1 + Q = E_R + E_0 \quad (1)$$

$$\vec{P}_1 = \vec{P}_0 + \vec{P}_R \quad (2)$$

where the E's and P's are kinetic energy and momentum, respectively. Using the conservation relations and solving the momentum triangle<sup>11</sup> it is found that

$$E_R = \frac{M_1}{M_R} E_1 + \frac{M_0}{M_R} E_0 - 2 \cos \theta \frac{\sqrt{M_1 M_0}}{M_R} \sqrt{E_1 E_0} + \delta_{\text{rel}} \quad (3)$$

where

$$\delta_{\text{rel}} \approx \frac{1}{2M_R c^2} \left( E_1^2 + E_0^2 - E_R^2 - \cos \theta \sqrt{M_1 M_0} \sqrt{E_1 E_0} \left( \frac{E_1}{M_1} + \frac{E_0}{M_0} \right) \right) \quad (4)$$

This equation contains  $E_R$  but sufficient accuracy is obtained if the nonrelativistic expression is used:

11.7. Spectral Properties

### The Spectrum

The type of transition observed in this case is illustrated in

Figure 2 (Infrared region).

The form of the spectrum of energy and momentum transfer

$$(1) \quad \vec{p} = \vec{p}' + \vec{p}''$$

$$(2) \quad \vec{p} = \vec{p}' + \vec{p}''$$

where the first two terms are the initial and final momenta, respectively, and the third term is the momentum transfer.

It is found that

$$(3) \quad \vec{p} = \vec{p}' + \vec{p}''$$

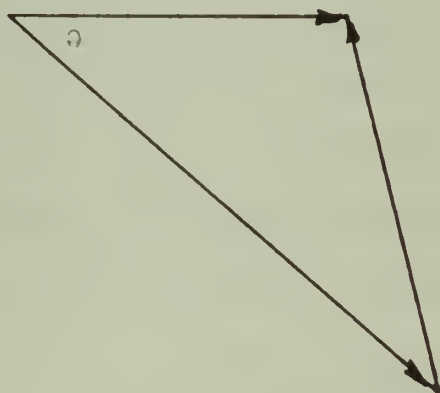
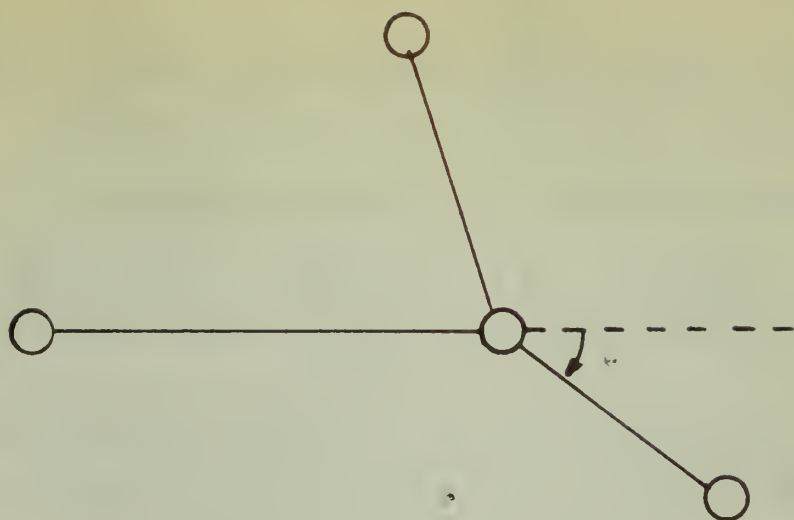
where

$$\vec{p} = \vec{p}' + \vec{p}''$$

(4)

This equation defines the initial and final momenta, respectively, and the third term is the momentum transfer.

It is found that





$$E_R = \frac{M_1}{M_R} E_1 + \frac{M_0}{M_R} E_0 - 2 \cos \theta \frac{\sqrt{M_1 M_0}}{M_R} \sqrt{E_1 E_0} \quad (5)$$

Combining (1) and (4), the following equation is obtained:

$$Q = \frac{M_R + M_0}{M_R} E_0 - \frac{M_R - M_1}{M_R} E_1 - 2 \cos \theta \frac{\sqrt{M_1 M_0}}{M_R} \sqrt{E_1 E_0} + \delta_{rel.} \quad (6)$$

This equation expresses  $Q$  in terms of the energies of the incident and output particles, which can be measured in each experiment.

To obtain the  $Q$ -value corresponding to the distance along the plate to the one-third height of each proton group, the following procedure is employed. The radius of curvature ( $\rho$ ) of the particle, corresponding to this distance, is obtained from the calibration curve described above. This is multiplied by the magnetic induction ( $B$ ) measured by the fluxometer to give the momenta  $B\rho$  of the particles. The relationship between  $B\rho$  and energy is<sup>11</sup>

$$E = m_0 c^2 \left[ \frac{1}{1 + \left( \frac{ZeB\rho}{m_0 c} \right)^2} - 1 \right]$$

where

$m_0$  = electron mass

$c$  = velocity of light

$Z$  = charge of particle in units of electronic charge

$B$  = magnet induction

$\rho$  = radius of curvature

$e$  = electronic charge



$$(2) \quad \frac{h^2}{2m} \frac{d^2 \psi}{dx^2} + V(x) \psi = E \psi$$

Combining (1) and (2), the following equation is obtained:

$$(3) \quad \frac{h^2}{2m} \frac{d^2 \psi}{dx^2} + V(x) \psi = E \psi$$

This equation represents a wave function of the incident and output particles, which can be measured in our experiment. To obtain the  $\psi$ -value corresponding to the distance along the plate to the one-half inch of each wave group, the following procedure is employed. The radius of curvature ( $\rho$ ) of the particle, corresponding to that distance, is obtained from the collision curve described above. This is multiplied by the magnetic field ( $B$ ) measured at the distance to give the momenta ( $p$ ) of the particles. The relationship between  $p$  and energy is

$$E = \frac{p^2}{2m}$$

where

- $m$  = electron mass
- $E$  = energy of light
- $p$  = change of radius in units of electronic charge
- $B$  = magnetic induction
- $\rho$  = radius of curvature
- $\psi$  = electronic charge

This equation may be expanded in powers of  $(B\rho)^2$ . The energies of protons, deuterons, tritons, and alpha particles have been tabulated by Enge<sup>11</sup> for values of  $B\rho$  from  $10^5$  to  $6.5 \times 10^5$  gauss centimeters. Using the value of input energy determined by elastic scattering and the output energy determined as described here, one can calculate the  $Q$ -value for each level. The calculations were greatly facilitated through the efforts of Mr. Pieter Mimmo, who programmed this calculation for the IBM 704 computer at M.I.T. The results were presented as a table of  $Q$ -values for each centimeter of plate distance. Linear interpolation was used to find the  $Q$ -value corresponding to each proton group.

#### Target Mass Analysis

In preparation for exposing the targets to the deuteron beam, it is essential to know the relative abundance of target nuclei and contaminants. This information is useful as an aid in assigning the proton groups observed in the analysis of the  $(d,p)$  reaction to the proper reaction. Mass analysis of the target is accomplished by elastically scattering incident protons from the target and analyzing the scattered protons with the broad-range spectrograph. Because each nucleus will absorb momentum from the incident beam in inverse proportion to its mass, it is possible to identify a proton group with the mass of the scattering nucleus. From the  $Q$ -equation with  $Q$  set equal to zero for elastic scattering, we obtain, ignoring the small relativistic correction  $\delta_{rel}$ :





$$m = \frac{m_p(E_0 + E_1) + 2 \cos \theta m_p \sqrt{E_1 E_0}}{E_0 - E_1}$$

Where

$m$  = mass of scattering nucleus

$m_p$  = mass of proton

$E_0$  = energy of scattered proton

$E_1$  = energy of incident proton

$\theta$  = observation angle (laboratory system).

Figure 4 shows the results of a scattering mass analysis performed on the target used in this work. A beam of 6.03-Mev protons was used, and the scattered particles were analyzed at a reaction angle of 130 degrees. The elements present in the target can be attributed to the following:

Tantalum = boat material used in evaporation

Silver = evaporated material ( $Ag_4P_{207}$ )

Phosphorus = evaporated material

Sodium = contaminant in evaporated material

Oxygen = Formvar backing; evaporated material

Nitrogen = Formvar backing; contaminant in evaporated material

Carbon = Formvar backing

Chlorine, silicon, and potassium = materials previously evaporated and not completely purged from system; contaminants in evaporated material.

$$n = \frac{2.5 \times 10^4 \sqrt{1 + 2 \sin^2 \theta} + 1}{10^4}$$

$$10^4 - 1$$

where

$n$  = mass of suspended material

$10^4$  = mass of water

$2.5 \times 10^4$  = density of material in g/cm<sup>3</sup>

$10^4$  = density of liquid medium

$\theta$  = observation angle (degrees)

Figure 1 shows the results of a series of measurements

performed on the target with the target at 0.03-100

degrees was used, and the resulting particles were analyzed as a

function of the angle of observation. The elements present in the target

can be determined by the following:

Carbon = mass of carbon in the target

Hydrogen = mass of hydrogen in the target

Helium = mass of helium in the target

Neon = mass of neon in the target

Argon = mass of argon in the target

Krypton = mass of krypton in the target

Xenon = mass of xenon in the target

Radiation = mass of radiation in the target

Calculation of the mass of the target is given by

mass of target = mass of material in target

mass of material in target = mass of material in target



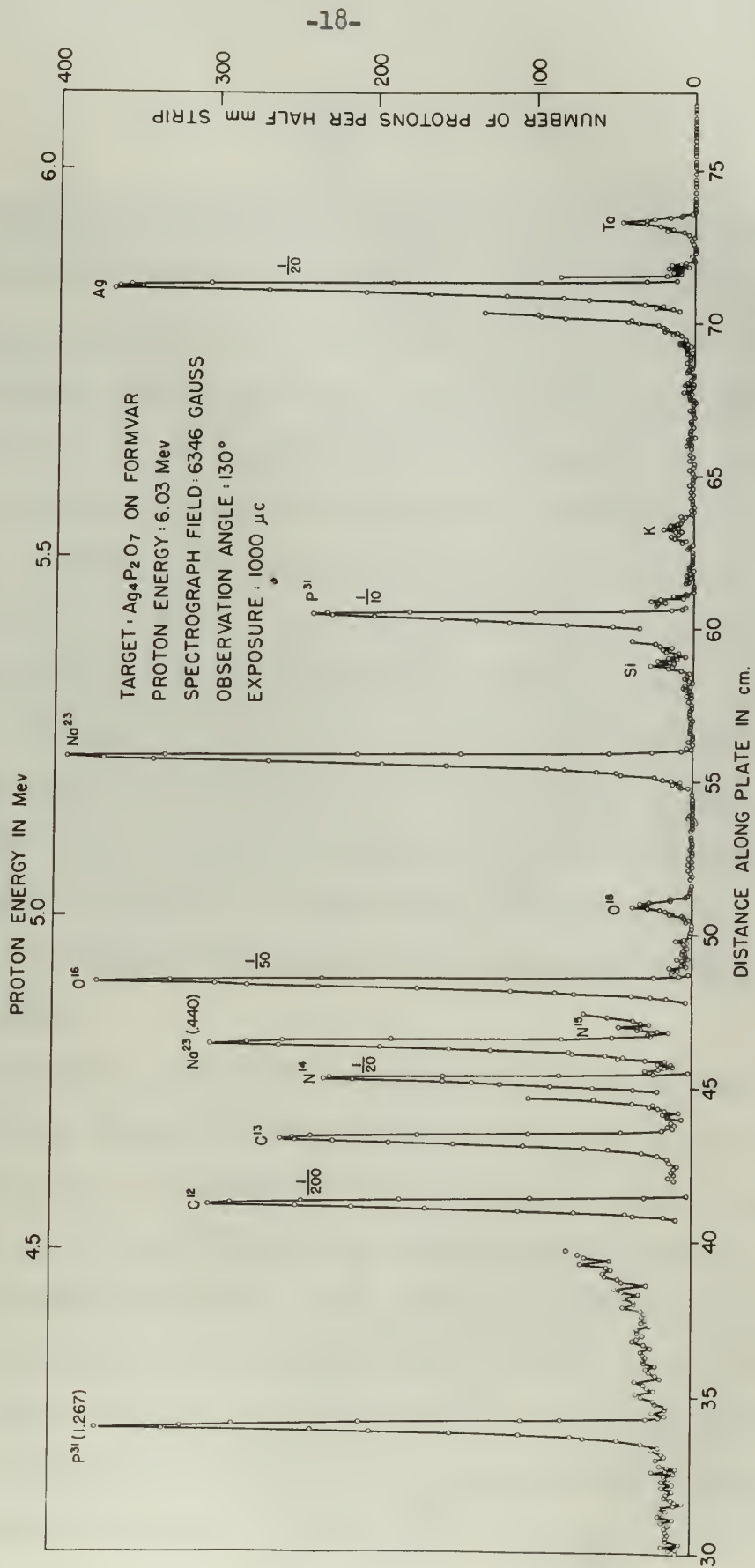


Figure 4.



The unusually high concentrations of sodium and nitrogen can be explained by noting the procedure for synthesizing silver pyrophosphate. This consists of combining silver nitrate and sodium pyrophosphate, which could explain the presence of these impurities. Inquiries at all leading chemical supply firms failed to produce a source of this compound with a high degree of purity. Attempts were made to evaporate metallic phosphates, which are available with a high degree of purity. It was found that it was not possible to get sufficient amounts of phosphorus nuclei onto the target, even with targets of 15-kev thickness. In particular, lead phosphate and cadmium phosphate were evaporated. A mass analysis of these targets showed large amounts of the metallic nuclei but extremely small quantities of phosphorus. Further study of this phenomenon is indicated to determine what happens to the phosphorus in the evaporation process.

Although the amounts of contaminant nuclei in the target were significantly higher than can normally be tolerated, it was decided to proceed with the analysis of the (d,p) reaction. The contaminant elements that would be expected to give appreciable yields are carbon, nitrogen, oxygen, and sodium. Since these elements have all been previously studied in this laboratory, it was felt that no difficulty would be experienced in picking out the proton groups from these contaminants. It is possible to distinguish unambiguously the proton groups ascribed to the  $P^{31}(d,p)P^{32}$  reaction from the proton

The unusually high concentration of sodium and nitrogen can be explained by noting the procedure for synthesizing silver pyrophosphate. This consists of combining silver nitrate and sodium pyrophosphate, which would explain the presence of these elements. In addition, at all leading chemical supply firms failed to produce a source of this compound with a high degree of purity. Attempts were made to evaporate anhydrous phosphates, which are available with a high degree of purity. It was found that it was not possible to get sufficient amounts of phosphorus without using the target, even with targets of low thickness. In particular, lead phosphates and calcium phosphates were evaporated. A mass analysis of these targets showed large amounts of the material used and extremely small quantities of phosphorus. Further study of this phenomenon is indicated so that it may be possible to use phosphorus in the evaporation process.

Although the amount of phosphorus used in the target was significantly higher than can normally be obtained, it was decided to proceed with the analysis of the (5,0) reaction. The contaminant elements that would be expected to give spectroscopic signals are carbon, nitrogen, oxygen, and sodium. Other elements known to be present previously studied in this laboratory, it was felt that it might only be expected to be detected in the reaction from these contaminants. It is possible in this reaction to detect the proton groups excited to the  $2p^2(4,2)$  transition from the ground



groups arising from the contaminant elements by noting the relative shift in proton energy as a function of observation angle.

### The (d,p) Reaction

A deuteron beam of 6.01 Mev was used to bombard the target. The energy of the incident deuterons was determined by measuring the energy of the deuterons elastically scattered from the phosphorus. Solving the Q- equation for  $E_1$ , we obtain:

$$E_1 = \frac{M_R + M_0}{M_R - M_1} E_0 - \frac{2 \cos \theta \sqrt{M_1 M_0 E_1}}{M_R - M_1} + \delta_{\text{rel}} \frac{M_R}{M_R - M_1}$$

This equation can be solved by successive approximations for  $E_1$  in terms of the energy of the scattered deuterons.

The following exposures were used at the various angles of observation:

30 degrees	-	500 $\mu$ c
50 degrees	-	1500 $\mu$ c
70 degrees	-	1000 $\mu$ c
90 degrees	-	1000 $\mu$ c

The charged-particle beam, which is incident on the target, is measured and integrated by a combined sensitive microammeter and current integrater designed by Enge.<sup>12</sup> A 300-volt bias is employed in the target chamber to obviate the possibility of measuring false currents produced by secondary electrons. An additional expo-

groups arising from the corresponding elements of both the relative shift in proton energy as a function of concentration angle.

### The (4,2) transition

A detector beam of 4.02 mm was used to detect the signal. The width of the incident beam was determined by measuring the energy of the incident electrons scattered from the 4000-ohm resistor. Solving the 0- equation for  $\theta_1$  we obtain:

$$\theta_1 = \frac{\theta_0 + \theta_2}{2} - \frac{2 \cos \theta_0 \sqrt{1 - \cos^2 \theta_0}}{1 - \cos^2 \theta_0} + \frac{1}{2} \ln \frac{1 + \cos \theta_0}{1 - \cos \theta_0}$$

This equation may be solved by successive approximations for  $\theta_1$ . In terms of the energy of the scattered electrons.

The following experiments were made at two various angles of

observation:

- 30 degrees - 200 Hz
- 50 degrees - 1000 Hz
- 70 degrees - 1000 Hz
- 90 degrees - 1000 Hz

The energy-distribution beam, which is located on the beam, is separated and detected by a constant sensitive detector. The current detector is of the 100-ohm type. It is placed in the beam of the detector for the purpose of measuring the current. The detector is connected to a constant current source.

sure of one-tenth the above values was made at each angle in order that the more intense proton groups, particularly those from carbon and oxygen, could be counted. Since the  $Q$ -values for the  $C^{12}(d,p)C^{13}$  and  $O^{16}(d,p)O^{17}$  reactions are well known, these groups serve as a good check on the input energy and also on the spectrograph field settings.

A typical spectrum of the proton groups is shown in Figure 5. It is noted that some of the groups are not resolved from contaminant groups, but these were all observed on at least one other exposure at a different observation angle where the relative shift in proton energy was sufficient to allow resolution of the phosphorus groups from those that were due to contaminants.

sure of one-kind the above values are made at each step in order that the more intense regions, especially those from regions

and regions, could be located, since the regions for the

$C_{12}(4,9)C_{12}(1,9)$  and  $C_{12}(1,9)C_{12}(4,9)$  regions are well known, these regions

serve as a first check on the input matrix and also on the spectra-

these fields position.

A typical spectrum of the system given is shown in figure 2.

It is noted that one of the regions are not resolved from another.

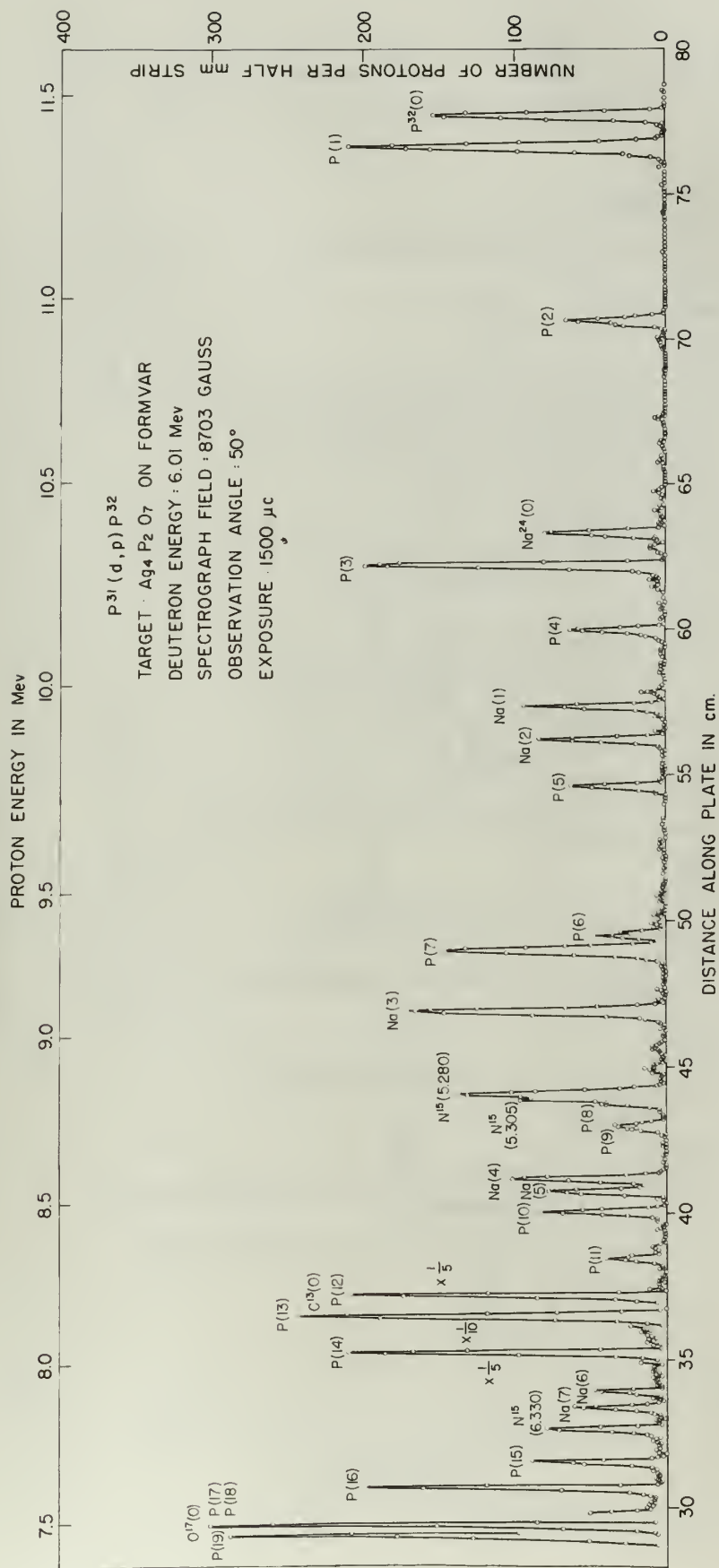
and, hence, the data are all covered on at least one other and

regions at a different observation angle where the relative shift in

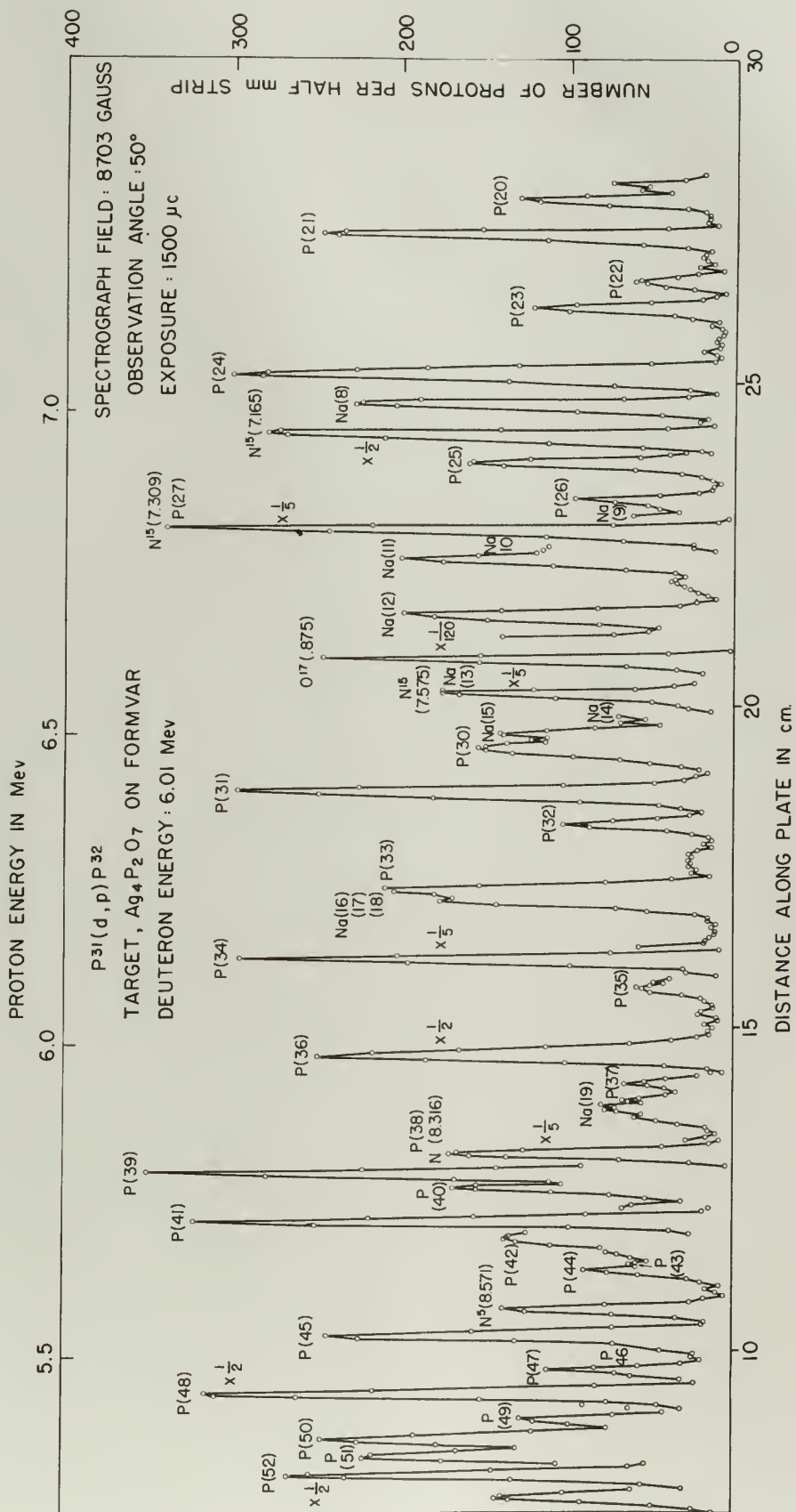
position enough are indicated to also resolution of the phosphorus

regions from those that were due to components.













#### IV. Results

##### The $P^{31}(d,p)P^{32}$ Reaction

The ground-state Q-value for the  $P^{31}(d,p)P^{32}$  reaction was determined as  $5.709 \pm 0.010$  Mev, which agrees within the stated errors with the value of  $5.704 \pm 0.003$  Mev reported by Van Patter et al.<sup>13</sup> Moreover, fifty-two proton groups were assigned to excited levels in  $P^{32}$  by virtue of their energy shift with observation angle. The results are summarized in Table I and Figure 6. The results of Van Patter et al, and those of Dalton et al.<sup>14</sup> are included for purposes of comparison. It is seen that agreement of the present work with that previously reported is quite good.

##### The $Na^{23}(d,p)Na^{24}$ Reaction

Sufficient yield was obtained from the  $Na^{23}(d,p)Na^{24}$  reaction so that these results can also be presented for comparison with the work of Sperduto and Duechner.<sup>15</sup> A ground-state Q-value of  $4.731 \pm 0.010$  Mev was found for the  $Na^{23}(d,p)Na^{24}$  reaction in exact agreement with previous results. All nineteen of the excited levels reported earlier were confirmed in this investigation. These results are presented in Table II. It is somewhat surprising that there were no proton groups beyond the one corresponding to the 4.56-Mev level in  $Na^{24}$  which could be assigned to  $Na^{24}$ . It is probable that relatively weak sodium groups in this region may have been obscured at all observation angles by other more intense groups from  $P^{32}$  and

IV. Results

The  $P_{II}(q, \nu)$  reaction

The ground-state values for the  $P_{II}(q, \nu)$  reaction are determined as  $2.709 \pm 0.010$  MeV, which agrees within the stated errors with the value of  $2.708 \pm 0.008$  MeV reported by the authors of ref. 13. Moreover, fifty-two proton groups were assigned to excited levels in  $P_{II}$  by virtue of their energy shift with absorption angle. The results are summarized in Table I and Figure 6. The results of the present work, and those of ref. 13 are included for purposes of comparison. It is seen that agreement of the present work with that previously reported is quite good.

The  $^{23}S(q, \nu)$  reaction

Difficulties arise in assigning levels from the  $^{23}S(q, \nu)$  reaction so that these results can also be presented for comparison with the work of Gertsch and Macdonald.<sup>12</sup> A ground-state value of 4.111  $\pm 0.010$  MeV was found for the  $^{23}S(q, \nu)$  reaction in our first series with previous results. All members of the excited levels reported earlier were confirmed in this investigation. These results are presented in Table II. It is somewhat surprising that there were no other groups beyond the one corresponding to the 4.46-MeV level in  $^{23}S$  which could be assigned to  $^{23}S$ . It is probable that relatively weak neutron groups in this region may have been assigned as all observation angles of other more intense groups from  $P_{II}$  and

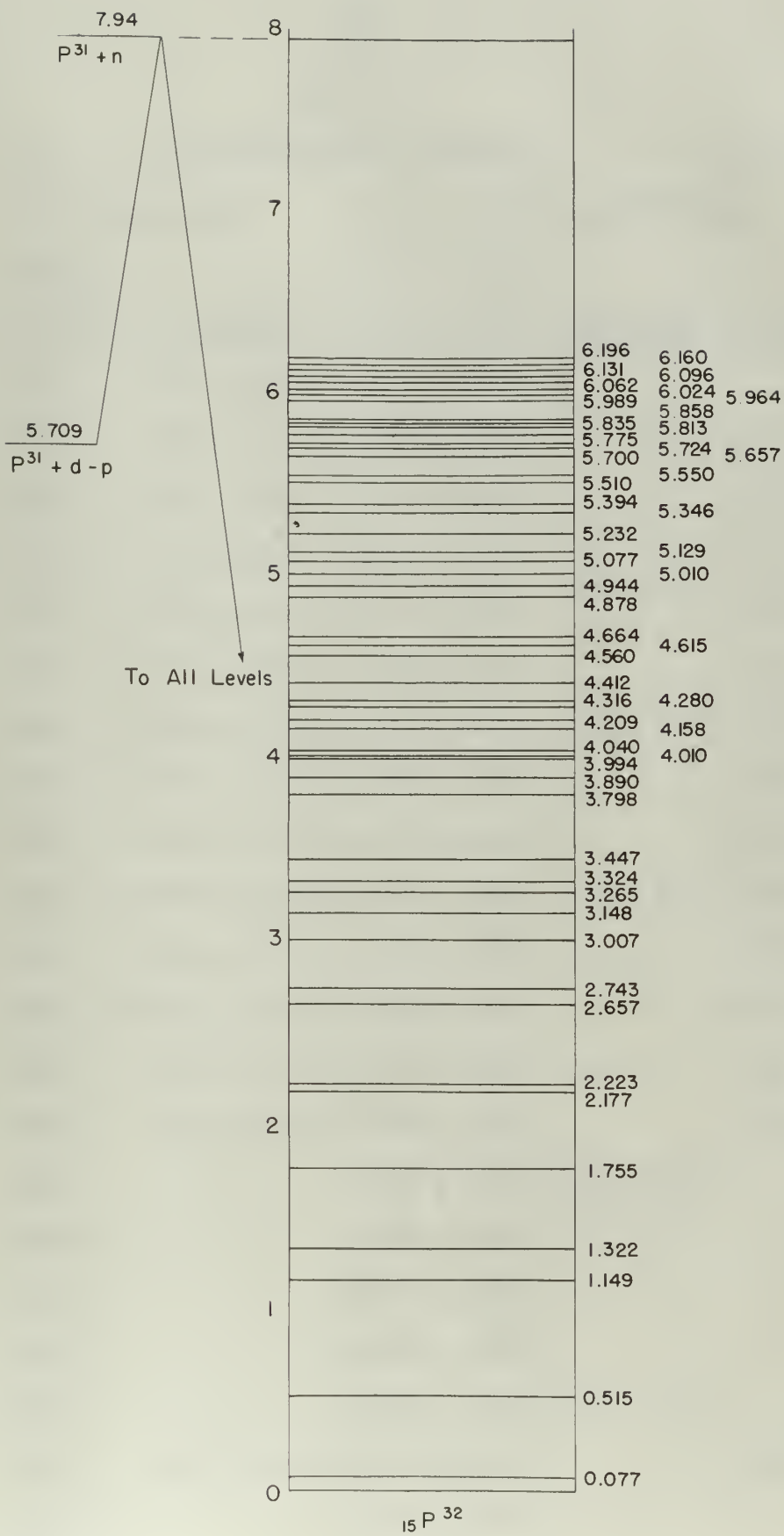






Table I  
The  $P^{31}(d,p)P^{32}$  Reaction

Q-Value (Mev)		Levels in $P^{32}$ (Mev)			
Level	Present Work $\pm 0.010$	Van Patter <sup>13</sup>	Present Work	Dalton <sup>14</sup>	Van Patter <sup>13</sup>
0	5.709	$5.704 \pm 0.003$	0	0	0
1	5.632	$5.627 \pm 0.003$	$0.077 \pm 0.003$		$0.077 \pm 0.002$
2	5.193	$5.189 \pm 0.010$	$0.516 \pm 0.003$		$0.515 \pm 0.005$
3	4.560	$4.550 \pm 0.007$	$1.149 \pm 0.003$		$1.154 \pm 0.007$
4	4.387	$4.388 \pm 0.007$	$1.322 \pm 0.003$		$1.316 \pm 0.008$
5	3.954	$3.954 \pm 0.007$	$1.755 \pm 0.004$		$1.750 \pm 0.009$
6	3.532	$3.527 \pm 0.003$	$2.177 \pm 0.004$		$2.177 \pm 0.009$
7	3.486	$3.477 \pm 0.008$	$2.223 \pm 0.004$		$2.227 \pm 0.009$
8	3.052	$3.054 \pm 0.006$	$2.657 \pm 0.005$		$2.650 \pm 0.008$
9	2.966	$2.962 \pm 0.006$	$2.743 \pm 0.005$		$2.742 \pm 0.008$
10	2.702	$2.705 \pm 0.008$	$3.007 \pm 0.005$		$2.999 \pm 0.010$
11	2.561	$(2.563 \pm 0.010)$	$3.148 \pm 0.005$		$(3.141 \pm 0.012)$
12	2.444	$2.445 \pm 0.006$	$3.265 \pm 0.005$		$3.259 \pm 0.009$
13	2.385	$2.386 \pm 0.006$	$3.324 \pm 0.005$		$3.318 \pm 0.009$
14	2.262		$3.447 \pm 0.005$	$3.45 \pm 0.10$	
15	1.911		$3.798 \pm 0.006$		
16	1.819		$3.890 \pm 0.006$		
17	1.715		$3.994 \pm 0.006$		
18	1.699		$4.010 \pm 0.006$		
19	1.669	$1.672 \pm 0.005$	$4.040 \pm 0.006$		$4.032 \pm 0.009$
20	1.551		$4.158 \pm 0.006$		

Table I  
The  $\chi^2$  function

(level in %) (level in %)

level	Present work	Present work	Present work	Present work
0	2.708	2.701 ± 0.001	0	0
1	2.232	2.227 ± 0.001	0.011 ± 0.001	0.011 ± 0.001
2	2.193	2.189 ± 0.001	0.016 ± 0.001	0.016 ± 0.001
3	1.250	1.250 ± 0.001	1.116 ± 0.001	1.116 ± 0.001
4	1.387	1.388 ± 0.001	1.325 ± 0.001	1.316 ± 0.001
5	3.221	3.221 ± 0.001	1.325 ± 0.001	1.320 ± 0.001
6	3.235	3.251 ± 0.001	5.111 ± 0.001	5.111 ± 0.001
7	3.186	3.111 ± 0.001	5.523 ± 0.001	1.521 ± 0.001
8	3.082	3.021 ± 0.001	5.621 ± 0.001	5.620 ± 0.001
9	5.464	5.462 ± 0.001	5.113 ± 0.001	5.112 ± 0.001
10	5.106	5.102 ± 0.001	3.021 ± 0.001	5.220 ± 0.001
11	5.251	(5.252 ± 0.010)	3.119 ± 0.001	(3.112 ± 0.012)
12	5.111	5.112 ± 0.001	3.522 ± 0.001	3.522 ± 0.001
13	5.382	5.320 ± 0.001	3.321 ± 0.001	3.320 ± 0.001
14	5.525	3.111 ± 0.001	3.112 ± 0.001	3.112 ± 0.001
15	1.211	3.128 ± 0.001		
16	1.819	2.112 ± 0.001		
17	1.112	3.211 ± 0.001		
18	1.622	1.110 ± 0.001		
19	1.662	1.012 ± 0.001		
20	1.221	1.128 ± 0.001		

21	1.499	$1.497 \pm 0.008$	$4.209 \pm 0.006$	$4.207 \pm 0.010$
22	1.428		$4.280 \pm 0.006$	
23	1.393		$4.316 \pm 0.006$	
24	1.296		$4.412 \pm 0.006$	$4.43 \pm 0.1$
25	1.149		$4.560 \pm 0.007$	
26	1.094		$4.615 \pm 0.007$	
27	1.045		$4.664 \pm 0.007$	
28	0.831		$4.878 \pm 0.007$	$4.90 \pm 0.07$
29	0.764		$4.944 \pm 0.007$	
30	0.699		$5.010 \pm 0.007$	
31	0.632		$5.077 \pm 0.007$	$5.11 \pm 0.1$
32	0.580		$5.129 \pm 0.007$	
33	0.476		$5.232 \pm 0.007$	
34	0.362		$5.346 \pm 0.007$	$5.37 \pm 0.07$
35	0.314		$5.394 \pm 0.007$	
36	0.199		$5.510 \pm 0.006$	
37	0.159		$5.550 \pm 0.008$	
38	0.052		$5.657 \pm 0.008$	$5.75 \pm 0.1$
39	0.009		$5.700 \pm 0.008$	
40	-0.016		$5.724 \pm 0.008$	
41	-0.067		$5.775 \pm 0.008$	
42	-0.104		$5.813 \pm 0.008$	
43	-0.126		$5.835 \pm 0.008$	$5.82 \pm 0.07$
44	-0.150		$5.858 \pm 0.008$	





45	-0.255	5.964 $\pm$ 0.008	
46	-0.280	5.989 $\pm$ 0.008	
47	-0.316	6.024 $\pm$ 0.008)	
48	-0.354	6.062 $\pm$ 0.008)	
49	-0.388	6.096 $\pm$ 0.008)	6.09 $\pm$ 0.07
50	-0.422	6.131 $\pm$ 0.008)	
51	-0.452	6.160 $\pm$ 0.008)	
52	-0.487	6.196 $\pm$ 0.008	6.20 $\pm$ 0.1

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		222.0 - 0.322	-89
		222.0 - 0.322	-90
		222.0 - 0.322	-91
		222.0 - 0.322	-92
		222.0 - 0.322	-93
		222.0 - 0.322	-94
		222.0 - 0.322	-95
		222.0 - 0.322	-96
		222.0 - 0.322	-97
		222.0 - 0.322	-98
		222.0 - 0.322	-99
		222.0 - 0.322	-100

Table II  
The  $\text{Na}^{23}(\text{d}, \text{p})\text{Na}^{24}$  Reaction

Level	Q-Value (Mev)		Levels in $\text{Na}^{24}$ (Mev)	
	Present Work $\pm 0.010$	Sperduto and Buechner	Present Work	Sperduto and Buechner
0	4.731	$4.731 \pm 0.007$	0	0
1	4.260	$4.259 \pm 0.007$	$0.471 \pm 0.003$	$0.472 \pm 0.008$
2	4.169	$4.167 \pm 0.007$	$0.562 \pm 0.003$	$0.564 \pm 0.008$
3	3.394	$3.390 \pm 0.006$	$1.337 \pm 0.003$	$1.341 \pm 0.008$
4	2.887	$2.887 \pm 0.006$	$1.844 \pm 0.004$	$1.844 \pm 0.008$
5	2.845	$2.847 \pm 0.006$	$1.886 \pm 0.004$	$1.884 \pm 0.008$
6	2.262	$2.267 \pm 0.006$	$2.469 \pm 0.004$	$2.464 \pm 0.008$
7	2.167	$2.170 \pm 0.006$	$2.564 \pm 0.005$	$2.561 \pm 0.008$
8	1.324	$1.322 \pm 0.005$	$3.407 \pm 0.005$	$3.409 \pm 0.008$
9	1.143	$1.149 \pm 0.006$	$3.588 \pm 0.006$	$3.582 \pm 0.009$
10	1.104	$1.108 \pm 0.006$	$3.627 \pm 0.006$	$3.623 \pm 0.009$
11	1.083	$1.083 \pm 0.006$	$3.648 \pm 0.006$	$3.648 \pm 0.009$
12	0.989	$0.993 \pm 0.005$	$3.742 \pm 0.006$	$3.738 \pm 0.008$
13	0.890	$0.881 \pm 0.005$	$3.841 \pm 0.006$	$3.850 \pm 0.008$
14	0.825	$0.832 \pm 0.005$	$3.906 \pm 0.006$	$3.899 \pm 0.008$
15	0.802	$0.802 \pm 0.005$	$3.929 \pm 0.006$	$3.929 \pm 0.008$
16	0.544	$0.547 \pm 0.005$	$4.187 \pm 0.006$	$4.184 \pm 0.008$
17	0.529	$0.529 \pm 0.005$	$4.202 \pm 0.006$	$4.202 \pm 0.008$
18	0.506	$0.512 \pm 0.005$	$4.225 \pm 0.006$	$4.219 \pm 0.008$
19	0.178	$0.173 \pm 0.005$	$4.563 \pm 0.007$	$4.558 \pm 0.009$

Table II  
The  $\chi^2$  test of the hypothesis

Index	Theoretical value	Observed value	The $\chi^2$ test of the hypothesis	
			Observed value	Theoretical value
0	0.171	0.171	0.171	0
1	0.250	0.250	0.250	0.171
2	0.169	0.169	0.169	0.171
3	0.334	0.334	0.334	0.171
4	0.007	0.007	0.007	0.171
5	0.017	0.017	0.017	0.171
6	0.255	0.255	0.255	0.171
7	0.157	0.157	0.157	0.171
8	0.357	0.357	0.357	0.171
9	0.173	0.173	0.173	0.171
10	0.170	0.170	0.170	0.171
11	0.093	0.093	0.093	0.171
12	0.090	0.090	0.090	0.171
13	0.090	0.090	0.090	0.171
14	0.092	0.092	0.092	0.171
15	0.090	0.090	0.090	0.171
16	0.274	0.274	0.274	0.171
17	0.259	0.259	0.259	0.171
18	0.200	0.200	0.200	0.171
19	0.173	0.173	0.173	0.171



the other contaminants, since the peak density is so high in this portion of the spectrum.

### Errors

The errors quoted in the Q-values and excitation energies are actually a measure of the uncertainties in the measurements of the energy involved. These uncertainties may be separated into two types: one random in nature, and the other systematic. A detailed examination of the uncertainty of each level has not been carried out, but the general effects have been noted, and it is felt that the probable error assigned is a generous estimate of the actual errors.

Q-Value Errors. The factors that may contribute to the random errors are:

1. The spread in energy of the incident particles resulting from finite slit widths;
2. The spread in energy of the emergent particles from variations in target thickness.
3. Failure to keep the fluxmeter pip centered on the oscilloscope.

Now, since each Q-value is the mean of at least two and usually four independent measurements, an estimate of the random error was obtained by calculating the standard error

$$\sigma = \left[ \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n(n-1)} \right]^{\frac{1}{2}}$$

the other components, since the first quantity is so high in this portion of the spectrum.

# Errors

The errors caused by the frequency and distribution weights are actually a measure of the uncertainty in the measurements of the energy involved. These uncertainties may be separated into two types: one known in advance, and the other systematic. A detailed examination of the uncertainty of each level has not been carried out, but the general effects have been noted, and it is this that has enabled error analysis to be carried out in a systematic manner of the actual errors.

## Systematic Errors and their contribution to the

random errors due to:

1. The spread in energy of the incident particles existing from kinetic effects;
2. The spread in energy of the scattered particles from variations in target thickness.
3. Variations in the thickness of the foil contained in the collimator.

Now, since each value is the mean of at least two and usually four independent measurements, an estimate of the random error was obtained by calculating the standard error

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n(n-1)}}$$

where

- $\bar{x}$  = mean value of  $Q$  for each level
- $x_i$  = individual  $Q$ -value measurements
- $n$  = number of observations.

This procedure resulted in an average value of 2.0 kev for the standard error, and this is taken as an estimate of the random error.

The systematic errors are the more significant ones, and in general they are a function of the energy of the emergent particles. These errors may be due to

1. The calibration error of the magnetic spectrograph, which includes the error in positioning the polonium wire and the uncertainty in the  $H_0$  value of the polonium alpha particles.

2. Uncertainties in determining the one-third height position of the proton peak, and the validity of using the one-third height as a measure of the proton energy. This error also includes the uncertainty in measuring the distance from the strip counted in the microscope to the index mark.

3. Since the energies of the elastically scattered deuterons used for input energy determination and of the protons from the (d,p) reaction are measured after passing through the target, there is an error in the  $Q$ -value determination from the difference in energy losses suffered by protons and deuterons. A correction for this effect has been applied to the  $Q$ -values, but it is difficult to analyze the effect precisely so that some error is introduced.



where

- $x$  = mean value of  $i$  for each level
- $x_i$  = individual  $i$ -value measurement
- $n$  = number of observations

This procedure resulted in an average value of 2.0 for the random error, and this is taken as the estimate of the random error. The systematic errors are the more difficult ones, and in general they are a function of the nature of the systematic variables. These errors may be due to

1. The estimation error in the systematic measurement, which includes the error in positioning the detector and the uncertainty in the value of the potential along the detector.
2. Uncertainties in determining the one-third depth position of the proton peak, and the validity of using the one-third depth as a measure of the proton energy. This error also includes the uncertainty in measuring the distance from the start counted in the microscope to the index mark.

3. Since the energies of the classically scattered deuterons used for target energy determination are of the order from the (d,p) reaction are measured after passing through the target, there is an error in the energy determination from the difference in energy losses suffered by protons and deuterons. A correction for this effect has been applied to the results, but it is difficult to assign the effect precisely to what some error is introduced.



The estimates for each of these errors is given in the following table:

Table III  
System Errors in Q-Values  
(in percent of proton energy)

1. Calibration error	0.04
2. Uncertainty in position of one-third height	0.03
3. Energy loss in target	0.03

Square root of sum of squares  $\approx$  0.06 percent.

The total error is then taken as the sum of the random and systematic errors. Thus, for the ground state at 30 degrees,

$$E_p = 11.6 \text{ Mev}$$

$$\text{Random error} = 2.0 \text{ kev}$$

$$\text{Systematic error} = 6.9 \text{ kev}$$

$$\text{Total error} = 9.0 \text{ kev.}$$

In order to allow for possible sources of error that may have been overlooked in this analysis, it was decided to list an error of  $\pm 10$  kev for all Q-values.

The statistics for each of these errors is given in the following table:

Table I

Table I

System errors in analysis

(in percent of nominal quantity)

1. Calibration error	0.02
2. Uncertainty in position of measuring points	0.02
3. Energy loss in cables	0.02

Spurious root of sum of squares  $\approx 0.01$  percent.

The total error is then given in the sum of the principal and systematic errors. Thus, for one percent error in the sum of the principal and

systematic errors, there is one percent error in the sum of the principal and

$$E_p = 11.1 \text{ per}$$

$$\text{calibration error} = 1.0 \text{ per}$$

$$\text{systematic error} = 0.1 \text{ per}$$

$$\text{Total error} = 1.1 \text{ per}$$

In order to allow for possible errors in error loss may have

been overlooked in this analysis, it was decided to list an error of

$\pm 10$  per for all deviations.

Excitation Energy Errors. The excitation energy is found by subtracting the  $Q$ -value for each level from the ground-state  $Q$ -value. The systematic errors in the  $Q$ -values then tend to cancel out, particularly for levels corresponding to proton groups which appear on the same plate as those from the ground-state transition. The random error was calculated as for the  $Q$ -values by examination of the standard errors of the mean value and was again found to be about 2 kev. The systematic error was taken as 0.1 percent of the excitation energy.

The following is a summary of the results of the analysis of the data obtained from the experiments. The results are presented in the form of a table, which is divided into two main parts. The first part, which is headed "Results of the Analysis of the Data", contains a table of the results of the analysis of the data. The second part, which is headed "Conclusions", contains a summary of the conclusions drawn from the analysis of the data. The results of the analysis of the data are presented in the following table:

Parameter	Value
Mean value of the mean value	0.1
Standard error of the mean value	0.01
Mean value of the standard error	0.01
Standard error of the standard error	0.01

The results of the analysis of the data are presented in the following table:

Parameter	Value
Mean value of the mean value	0.1
Standard error of the mean value	0.01
Mean value of the standard error	0.01
Standard error of the standard error	0.01

The results of the analysis of the data are presented in the following table:

Parameter	Value
Mean value of the mean value	0.1
Standard error of the mean value	0.01
Mean value of the standard error	0.01
Standard error of the standard error	0.01



## V. Conclusions

The present investigation of the  $P^{31}(d,p)P^{32}$  reaction using the broad-range spectrograph has extended the information on the energy levels of  $P^{32}$  up to 4.56-Mev excitation. In addition, several levels that were missed in previous investigations have been observed. The existence of the low-lying state at an energy of 77 kev has been confirmed. This state can be explained as arising from the two different orientations of the unpaired s-proton hole relative to the angular momentum of the unpaired d-neutron. The very small splitting observed requires a comparatively weak spin exchange force even though the two particles, being in different orbital states, do not interact strongly. Because the lowest states would be those with parallel spins, the ground state of  $P^{32}$  should have spin 1 and the excited states, spin 2. The states in question are members of spin doublets, and the dependence of their reaction cross section on the spin  $J_B$  of the residual nucleus is predicted by stripping theory.

In the stripping theory developed by Butler<sup>3</sup>, the only term in the cross-section formula that depends on the spin of the final nucleus is  $(2J_B + 1)$ . We can thus use the intensities of the proton groups corresponding to the two levels to check on the spin assignments made from stripping theory;  $J_B = 1$  for the ground state and  $J_B = 2$  for the 77-kev level. These values lead to a predicted ratio

# 7. Conclusions

The present investigation of the  $^{21}\text{F}(p,\gamma)^{22}\text{Ne}$  reaction using the direct-range spectrograph has extended the information on the energy levels of  $^{21}\text{F}$  up to 4.5-MeV excitation. In addition, several levels that were missed in previous investigations have been observed. The existence of the low-lying states at an energy of 77 keV has been confirmed. This state can be considered as arising from the  $41\frac{1}{2}^-$  parent configuration of the reaction  $^{21}\text{F}(p,\gamma)^{22}\text{Ne}$  relative to the angular momentum of the reaction  $^{21}\text{F}(p,\gamma)^{22}\text{Ne}$ . The only well established observed reaction is comparatively weak, but excitation from even though the two particles, being in different orbital states, do not interfere strongly. However, the lowest states would be those with parallel spins, the ground state of  $^{22}\text{Ne}$  should have spin 1 and the excited states, spin 2. The states in question are members of spin doublets, and the dependence of their reaction cross sections on the spin of the residual nucleus is predicted by statistical theory.

In the statistical theory developed by Bohr, the only term in the cross-section formula that depends on the spin of the final nucleus is  $(2J_f + 1)$ . We now have the intensities of the proton groups corresponding to the two levels in which the spin analysis made from existing theory:  $J_f = 1$  for the ground state and  $J_f = 2$  for the 77-keV level. These ratios lead to a predicted ratio

$$\frac{(2J_B + 1)_1}{(2J_B + 1)_0} = \frac{5}{3} = 1.67.$$

The ratios of intensities found in this experiment were 30°:1.61; 50°:1.52; 70°:1.44; and 90°:1.41 for an average of 1.5 so that at least qualitative agreement is evidenced.

This investigation served to demonstrate that an accurate analysis of energy levels can be made in the presence of a relatively large number of contaminant nuclei, provided the masses of the contaminants are not too close to the mass of the nucleus being studied. This ability becomes increasingly important as more nuclei are investigated which must be used in compound form to facilitate vacuum evaporation of the material onto the Formvar backings.



$$\frac{1}{2} = \frac{1}{2} \cdot \frac{(1 + \frac{1}{2})}{(1 + \frac{1}{2})}$$

The values of the constants found in this experiment were  $30 \pm 1.0$ ,  $30 \pm 1.5$ ,  $30 \pm 1.5$ , and  $30 \pm 1.5$  for an average of  $1.5$  as listed at least qualitative agreement is obtained.

This investigation served to demonstrate that an accurate

analysis of energy levels can be made in the presence of a relatively large number of constant nuclei, provided the number of the non-constant nuclei are not too close to the value of the constant nuclei. This ability becomes increasingly important as more nuclei are investigated which must be used in compound form to facilitate the investigation of the material under the present conditions.

It is noted that the values of the constants found in this experiment were  $30 \pm 1.0$ ,  $30 \pm 1.5$ ,  $30 \pm 1.5$ , and  $30 \pm 1.5$  for an average of  $1.5$  as listed at least qualitative agreement is obtained. This investigation served to demonstrate that an accurate analysis of energy levels can be made in the presence of a relatively large number of constant nuclei, provided the number of the non-constant nuclei are not too close to the value of the constant nuclei. This ability becomes increasingly important as more nuclei are investigated which must be used in compound form to facilitate the investigation of the material under the present conditions.



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